THE SHAPE OF THE THROWOUT CRATER FROM THE EXPLOSION OF A CORD CHARGE IN A TWO-LAYER MEDIUM

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The problem concerning the explosion of a cord charge in the ground, with a hard upper layer, is considered in pulse formulation. A numerical solution is obtained and profiles are given for the throwout craters for several values of thickness of the layer and emplacement depth of the charge, in two limiting cases: when the resistance of the lower ground is negligibly small in comparison with the resistance of the layer and when the resistance of the ground and layer are quite close in magnitude.

Kuznetsov [1] described the following formulation of the problem of an explosion in the ground. The ground is simulated by a medium which is an ideal incompressible liquid at velocities greater than a certain critical value, and by an absolutely solid body at velocities having a lower value. In the pulsed formulation, the boundary of the throwout crater is defined as the streamline at which the velocity modulus is equal to the critical value.

In this formulation, we consider the two-dimensional problem of determining the shape of the throwout crater from the explosion of a cord charge emplaced at a depth h_1 in ground which is covered by an upper layer of more resistant material with thickness h_2; i.e., it has a higher critical velocity value. Figure 1 shows a section in the plane perpendicular to the charge. By virtue of symmetry, we can confine our consideration to the regions of flow lying in the right-hand half-plane. We shall assume the charge, emplaced at the point Q, to be a source of yield 2q. Along the boundary FD of the region D_z the velocity modulus is equal to c_2 and along the boundary CB it is c_1.

We introduce the complex flow potential $w = \varphi + i\psi$ and the dimensionless variables

$$z^* = x^* + iy^* = \frac{c_1 z}{q}, \quad w^* = \frac{w}{q}, \quad c_1{}^* = 1, \quad c_2{}^* = \frac{c_2}{c_1}$$

and we formulate the problem in the following form (we omit the asterisks in the dimensionless quantities for simplicity). It is required to find the boundary of the unknown region D_z, in which the analytic function w(z) is defined with the boundary conditions

$$\begin{array}{lll}
\text{at } AB & \operatorname{Re} w = 0, & \arg \dfrac{dz}{dw} = \dfrac{\pi}{2} \\
\text{at } BC & \operatorname{Im} w = 0, & \left|\dfrac{dz}{dw}\right| = 1 \\
\text{at } CD & \operatorname{Im} w = 0, & \arg \dfrac{dz}{dw} = \pi \\
\text{at } DF & \operatorname{Im} w = 0, & \left|\dfrac{dz}{dw}\right| = \dfrac{1}{c_2} \\
\text{at } FQ & \operatorname{Im} w = 0, & \arg \dfrac{dz}{dw} = -\dfrac{\pi}{2} \\
\text{at } QA & \operatorname{Im} w = 1, & \arg \dfrac{dz}{dw} = \dfrac{\pi}{2}
\end{array}$$

It can be seen that in the plane w, the region of flow is represented by a half-fringe ($\psi = 0$, $\varphi = 0$, $\psi = 1$) which, by means of the function

$$\omega = 1 / \operatorname{ch}(\pi w),$$

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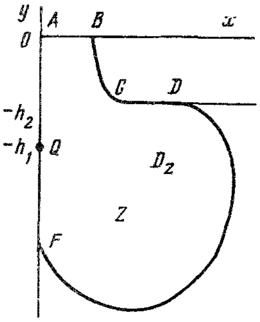

Fig. 1

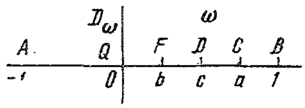

Fig. 2

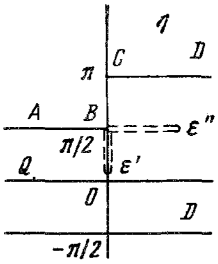

Fig. 3

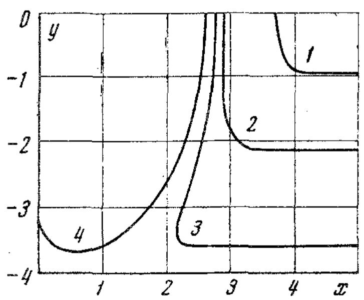

Fig. 4

is imaged at the upper half-plane (Fig. 2). In this region, we find the function

$$\Phi(\omega) = f[w(\omega)] = \ln\left(i\frac{dz}{dw}\right) \tag{1}$$

which, at the real axis, assumes the values

$$\begin{array}{lll} \text{at } B\infty A & \operatorname{Im}\Phi = 0 \\ \text{at } AQ & \operatorname{Im}\Phi = 0 \\ \text{at } QF & \operatorname{Im}\Phi = -\pi \\ \text{at } FD & \operatorname{Re}\Phi = \ln(c_2^{-1}) \\ \text{at } DC & \operatorname{Im}\Phi = \pi/2 \\ \text{at } CB & \operatorname{Re}\Phi = 0 \end{array} \tag{2}$$

Thus, we have obtained the Keldysh–Sedov problem [2]. In this paper, we consider the solution of this problem in two cases.

We put $c_2 = 0$ (this corresponds to a negligibly small resistance of the lower ground relative to the layer). Then the boundary of the region D_z (Fig. 1) is contracted to an infinitely distant point, but in the region D_ω (Fig. 2) the points F and D coincide ($b = c$). The problem is obtained concerning the breakdown of the layer with thickness h_2, which covers a liquid occupying the lower half-space. As a result of solving problems (1) and (2), we obtain the differential equation

$$\begin{aligned} i\frac{dz}{dw} = {} & [a^{-1/2} - (\omega-1)^{1/2}(\omega-a)^{-1/2}]\,[a^{-1/2} + (\omega-1)^{1/2}(\omega-a)^{-1/2}]^{-1} \\ & [(1-b)^{1/2}(a-b)^{-1/2} + (\omega-1)^{1/2}(\omega-a)^{-1/2}]^{3/2} \\ & [(1-b)^{1/2}(a-b)^{-1/2} - (\omega-1)^{1/2}(\omega-a)^{-1/2}]^{-3/2} \end{aligned} \tag{3}$$

[the constant $\Phi(\infty)$ is found from the condition that, at the point B, $\Phi = 0$]. It can be seen that in the solution of the problem, two parameters a and b occur, whose dependence on the quantities h_1 and h_2 is obtained after integration of Eq. (3) from the point Q to A and from C to B.

We consider now the effect of values of the parameters a and b on the flow geometry in the planes $\eta = \ln(dz/dw)$ (Fig. 3) and z (Fig. 4). In the plane η the region of flow is the quadrilateral QBCD when $b = b_* \equiv 5a/(9-4a)$.

In Fig. 4, curve 2 ($h_1 \equiv 3$, $h_2 = 2.2$) corresponds to this case; the horizontal section of this curve corresponds to the interface between the layer and the liquid. We note that at the surface $y = 0$, the curvature of this curve is equal to zero. When $b < b_*$, the flow region becomes the pentagon QE"BCD – curve 1 ($h_1 = 3$, $h_2 = 0.8$).

The horizontal indentation indicates that at the inside points, the velocity of the liquid reaches a value which is less than the critical velocity at the boundary. The boundary of the crater in this case becomes concave relative to the liquid. If $b > b_*$, we have the pentagon QBE'CD (vertical indentation) – curve 3 ($h_1 = 3$, $h_2 = 3.7$). On emerging at the surface, the boundary of the crater becomes convex relative to the liquid. For values of b close to a, the flow of liquid changes partially into the second sheet of a Riemann surface (in Fig. 5, $a = 0.2$; $b = 0.1999$; $h_1 = 0.68$; $h_2 = 1.1$; equipotentials are drawn by dashed lines). When $b = a$, the solution is shown by curve 4 ($h_1 = 3$).

Let us consider the case where the points D and C coincide (this means that the resistance of the surface and lower ground are quantities of a single order). In this case, after solving problems (1) and (2) with the condition that $\Phi(1) = 0$, we obtain

$$\begin{aligned} i\frac{dz}{dw} = c_2^{-1} & \left[\sqrt{\frac{1}{b}} + \sqrt{\frac{\omega-1}{\omega-b}}\right]\left[\sqrt{\frac{1}{b}} - \sqrt{\frac{\omega-1}{\omega-b}}\right]^{-1} \\ & \left[\left(\sqrt{\frac{\omega-1}{\omega-b}} + i\sqrt{\frac{1-a}{a-b}}\right)\left(\sqrt{\frac{\omega-1}{\omega-b}} - i\sqrt{\frac{1-a}{a-b}}\right)^{-1}\right]^{-\ln c_2/\pi i} \end{aligned}$$

Figure 6 shows the region of flow in the plane $\eta = \ln(dz/dw)$. The region is the quadrilateral QBDF with

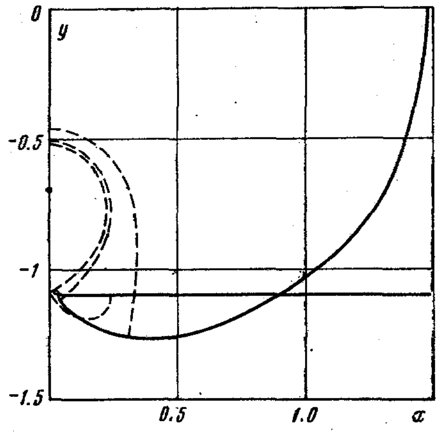

Fig. 5

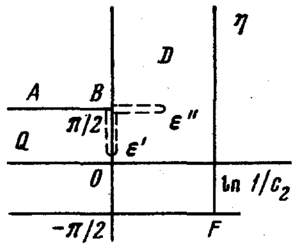

Fig. 6

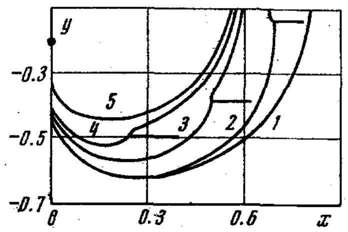

Fig. 7

$$a = a_* \equiv b\,(k^2 + 1)\,/\,(bk^2 + 1) \qquad (k = -\pi / \ln c_2)$$

When $a > a_*$, a horizontal indentation QE"BDF appears, and when $a \to 1$ the solution of this problem tends to the solution of the problem with a dimensionless critical velocity equal to c_2.

In Fig. 7, where the profiles of craters are shown for $h_1 = 0.2$, $c_2 = 0.5$ and different values of h_2, curves 2 ($h_2 = 0.13$) and 1 ($h_2 = 0$) correspond to this case. When $a < a_*$, a vertical indentation appears, QBE'DF, and when $a \to b$, we obtain the solution of the problem with a critical velocity equal to c_1 [curves 3 ($h_2 = 0.38$), 4 ($h_2 = 0.5$), and 5].

We note that at the point D where a rapid change of velocity occurs, the boundary of the crater fits with both sides along logarithmic spirals [3].

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